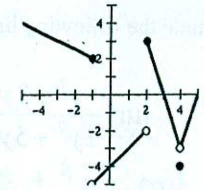
SCORE: _____ / 20 POINTS $\frac{3}{4}+$

1. NO CALCULATORS ALLOWED
2. UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS
3. SHOW PROPER CALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS

The graph of f is shown on the right. Evaluate the following limits. Write "DNE" if a limit does not exist. **SCORE: 3 / 3 PTS**



[a] $\lim\limits_{x\to 4}\dfrac{3x}{5-f(x)}$

$= \dfrac{\lim\limits_{x\to 4} 3x}{\lim\limits_{x\to 4} 5 - \lim\limits_{x\to 4} f(x)}$ (1)

$= \dfrac{3(\lim\limits_{x\to 4} x)}{5-(-3)} = \dfrac{3(4)}{5+3} = \dfrac{12}{8} = \boxed{\dfrac{3}{2}}$ (1)

[b] $\lim\limits_{x\to -1^+} f(x)$

$= \boxed{-5}$ (1)

Prove that $\lim\limits_{x\to 0} x^6\cos\dfrac{1}{x^3} = 0$. **SCORE: 3 / 3 PTS**

WE GOT $-1 \le \cos\frac{1}{x^3} \le 1$.

$-x^6 \le x^6\cos\frac{1}{x^3} \le x^6$. (1)

SO $\lim\limits_{x\to 0}(-x^6) = 0$ $\left(\frac{1}{2}\right)$

AND

$\lim\limits_{x\to 0}(x^6) = 0$. $\left(\frac{1}{2}\right)$

$\Rightarrow \lim\limits_{x\to 0} x^6\cos\frac{1}{x^3} = 0$ BY SQUEEZE THEOREM. (1)

If $\lim\limits_{r\to -1}\dfrac{4+ar-r^6}{1+r}$ exists, find the value of a. **SCORE: 0 / 2 PTS**

Using complete sentences and proper mathematical notation, write the formal definition of "vertical asymptote". **SCORE:** 0 / 2 PTS

Evaluate the following limits. Write "DNE" if a limit does not exist. **SCORE:** _____ / 7 PTS

[a] $\lim_{y \to -4} \frac{y^2+2y-8}{2y^2+5y-12}$

$$= \frac{\lim_{x\to 4} y^2+2y-8}{\lim_{x\to 4} 2y^2+5y-12}$$

$$= \frac{\lim_{x\to 4} y^2 + \lim_{x\to 4} 2y - \lim_{x\to 4} 8}{\lim_{x\to 4} 2y^2 + \lim_{x\to 4} 5y - \lim_{x\to 4} 12}$$

$$= \frac{(4)^2 + 2(4) - 8}{2(4)^2 + 5(4) - 12} = \frac{16+8-8}{32+20} = \frac{16}{52} = \boxed{\frac{4}{13}}$$

[b] $\lim_{b \to 3} \frac{b-\sqrt{b+6}}{6-2b} = \lim_{b\to 3} \frac{b-\sqrt{b+6}}{6-2b} \cdot \frac{b+\sqrt{b+6}}{b+\sqrt{b+6}}$

$$= \lim_{b\to 3} \frac{b^2 - \sqrt{b+6}^2}{(6-2b)(b+\sqrt{b+6})}$$

$$= \lim_{b\to 3} \frac{b^2-(b+6)}{(6-2b)(b+\sqrt{b+6})}$$

$$= \lim_{b\to 3} \frac{b^2-b-6}{(6-2b)(b+\sqrt{b+6})} \quad 1$$

$$= \lim_{b\to 3} \frac{\cancel{(6-2b)}(-\frac{1}{2}b-1)}{\cancel{(6-2b)}(b+\sqrt{b+6})} \quad *$$

$$= \frac{-1/2(3)-1}{3+\sqrt{9}} = \boxed{\frac{-5}{12}} \quad \frac{1}{2}$$

[c] $\lim_{t \to -5} \frac{\frac{6}{t-1} - \frac{4}{t+3}}{t^2+25}$

$$= \lim_{t\to -5} \frac{\frac{6(t+3)-4(t-1)}{(t-1)(t+3)}}{t^2+25}$$

$$= \lim_{t\to -5} \frac{\frac{(6t+18)-4t+4}{t^2+2t-3}}{t^2+25}$$

$$= \lim_{t\to -5} \frac{2t+22}{t^2+2t-3} \cdot \frac{1}{t^2+25}$$

$$= \lim_{t\to -5} \frac{2t+22}{(t^2+2t-3)(t^2+25)}$$

$$= \frac{2(-5)+22}{((-5)^2+2(-5)-3)((-5)^2+25)} = \frac{12}{(12)(50)} = \boxed{\frac{1}{50}} \quad 1$$

[d] $\lim_{x \to -2} f(x)$ where $f(x) = \begin{cases} 2x+1, & \text{if } x < -2 \\ x-1, & \text{if } -2 < x < 3 \\ 5-x, & \text{if } x > 3 \end{cases}$

$$\lim_{x\to -2^+} (5-x) = 5-(-2) = 7$$

$$\lim_{x\to -2^-} (x-1) = -2-1 = -3$$

SINCE $\lim_{x\to -2^+} f(x) \neq \lim_{x\to -2^-}$,

SO $\lim_{x\to -2} f(x)$ does not exist.